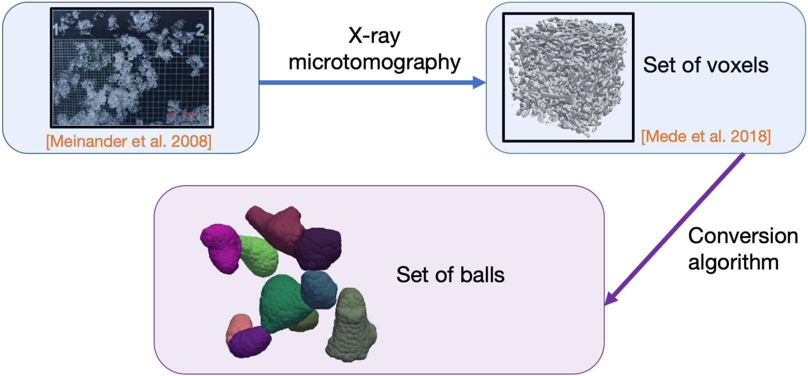
Exact and optimal conversion of a hole-free 2D digital object into a set of balls

Isabelle Sivignon

International Conference on Discrete Geometry and Mathematical Morphology,
Oct. 24-27, 2022, Strasbourg

Motivation

Conversion from one geometric model to another



[Mede et al. 2018]

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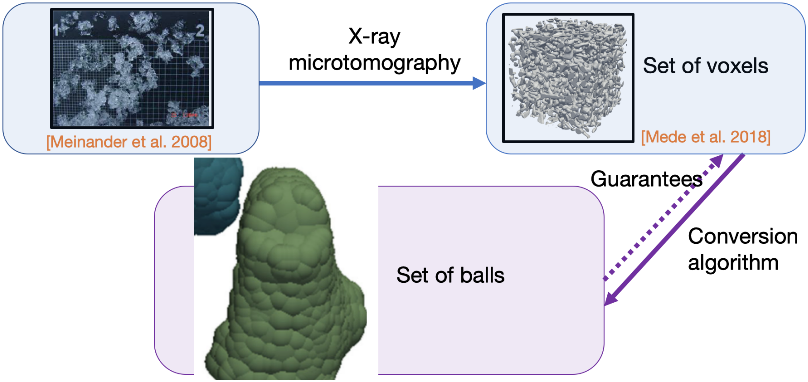

[Mede et al. 2018]

Set of pixels $\Longrightarrow$ set of balls

Digital object $=$ finite subset of $\mathbb{Z}^2$

Optimization problem

Given a $2\textsc{d}$ digital object S, compute a finite set of balls that:

- covers **all the points of** S and **no point of** $\mathbb{Z}^2 \backslash S$
- is of **minimum cardinality**.

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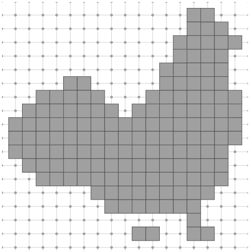
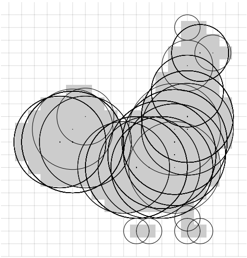


Distance tranformation
$+$
power map
$\Longrightarrow$

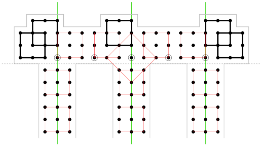


balls centered on $\mathbb{Z}^2$
$+$
not minimum

Summarized state of the art

Related problems

- NP-hard when the balls must be centered on $\mathbb{Z}^2$ [Coeurjolly, Hulin, S. 2008][Ragnemalm, Borgefors 93]



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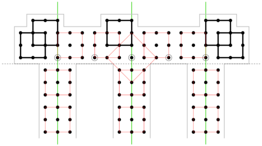
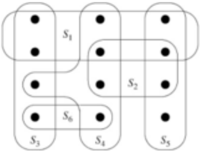
- NP-hard when the balls must be centered on $\mathbb{Z}^2$ [Coeurjolly, Hulin, S. 2008][Ragnemalm, Borgefors 93]
- Close to the set cover problem (also NP-hard): given a pair $(X, \mathcal{R})$, where X is a set of points and $\mathcal{R}$ is a family of subsets of X called *ranges*, find a minimum subset of $\mathcal{R}$ that covers all the points of X [Cormen, Leiserson, Rivest, 90]





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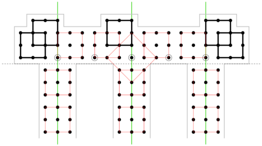

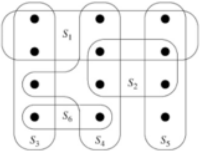

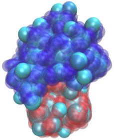

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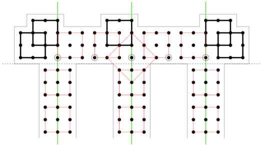
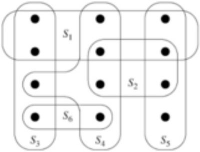
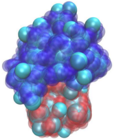
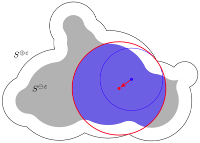
- inner/outer approximation of union of balls [Cazals et al. 2013]
- (δ, ϵ)-ball approximation [Nguyen 2018]: NP-hard in the general case, but polynomial time algorithm if shape has no hole





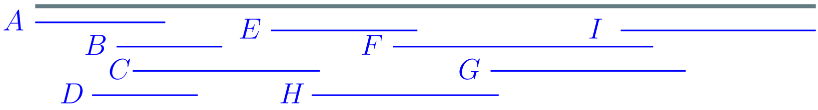




Interval cover

Specifications

Set of points $X \subset \mathbb{R}$
Set of ranges $\mathcal{R}$ = intervals
Assume that all intervals are maximal



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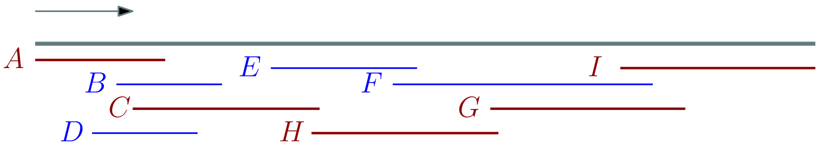

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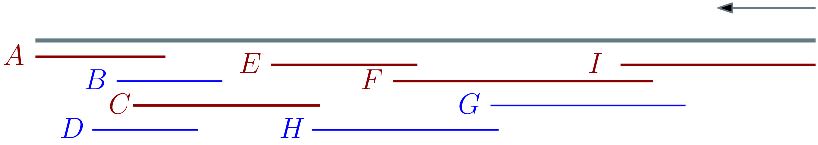
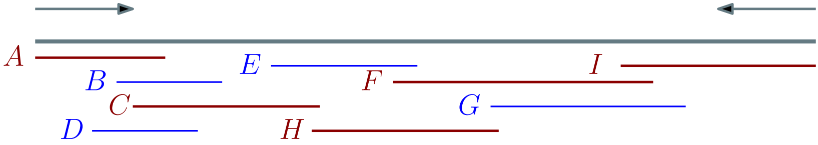


Interval cover - generalized algorithm



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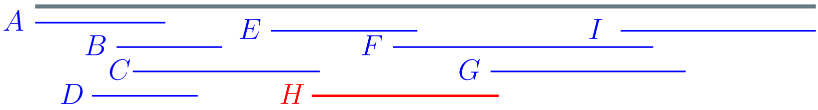

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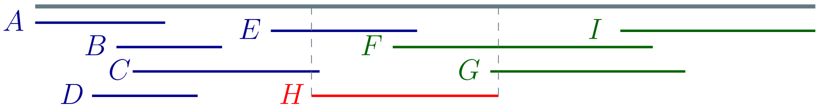

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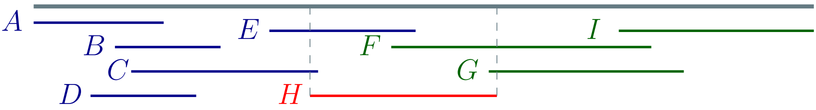

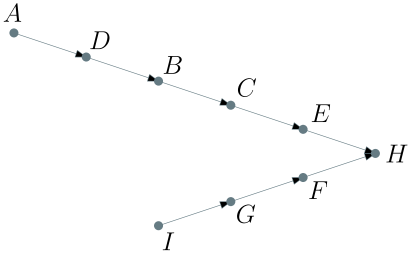

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  $A \to D$: A ends before D

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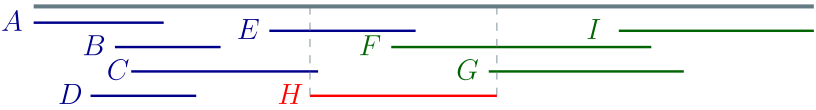

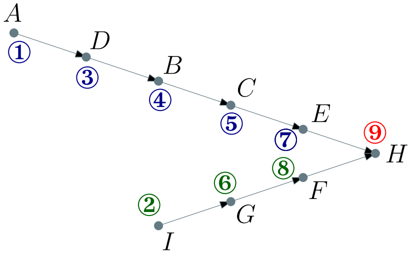

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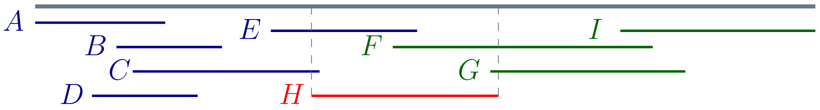

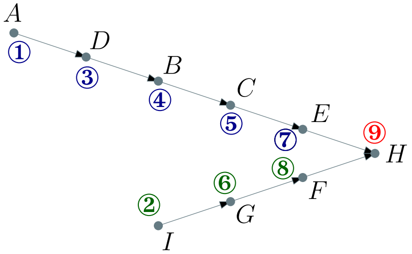

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Greedy Algorithm

$Cov \leftarrow \emptyset$;
$U \leftarrow X$ (set of uncovered points);
for $i \in \mathcal{I}$, *in topological order* **do**
  if i *is a maximal candidate for* U **then**
    $Cov \leftarrow Cov \cup \{i\}$;
    $U \leftarrow U \backslash i$;
return Cov

"i *maximal candidate for* U": i covers a point of U that is not covered by any interval "after" i

General set cover setting

Input/Output

Input: $(X, \mathcal{R})$ where X is a set of points, $\mathcal{R}$ a family of ranges (= subsets of X) and $\bigcup \mathcal{R} = X$
Output: a subset Cov of $\mathcal{R}$ such that Cov is a covering of X

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$\Longrightarrow$ which conditions for Cov to be a cardinal minimum covering ?

Optimal greedy algorithm ?

Sufficient conditions

If:

1. there exists a partial order $\preceq$ on $\mathcal{R}$ such that $(\mathcal{R}, \preceq)$ is **anti-arborescent**
2. $\forall x \in X$, the set $\{r \in \mathcal{R}, x \in r\}$ admits a **maximum** according to $\preceq$
3. $\forall r_1, r_2 \in \mathcal{R}$, $\forall r, r_1 \prec r \prec r_2$, $r_1 \cap r_2 \subseteq r$

then the greedy algorithm outputs a cardinal minimum covering of X.

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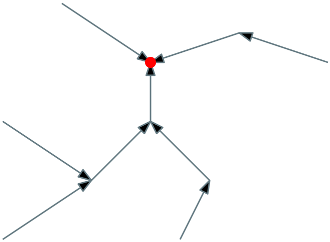

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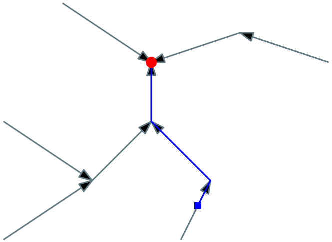

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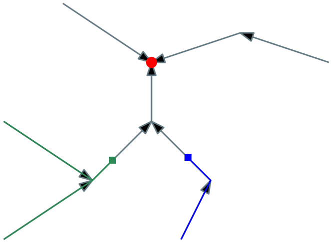

Back to the original problem

Problem

Given a 2D digital object S, compute a finite set of balls that:

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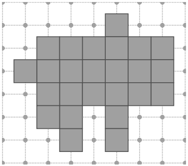

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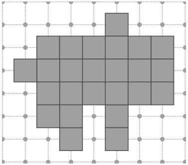

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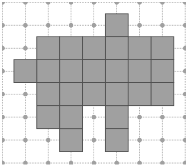
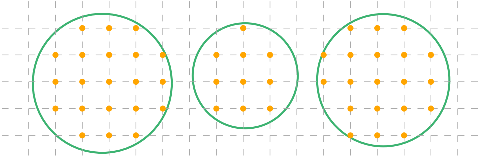
- **Digital ball** b = subset of $\mathbb{Z}^2$ for which there exists a ball $\mathscr{b}$ such that $Dig(\mathscr{b}) = \mathring{\mathscr{b}} \cap \mathbb{Z}^2 = b$



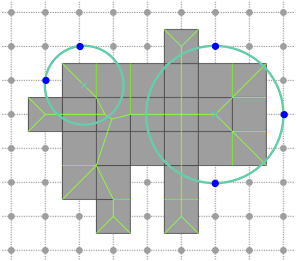


Grabbing maximal digital balls

Property

For any maximal digital ball b included in S, there exists a ball $\mathscr{b}$ such that $Dig(\mathscr{b}) = b$ and $\mathscr{b}$ has at least two points of $\mathbb{Z}^2 \backslash S$ on its boundary.

$\Longrightarrow$ compute the cropped Voronoi diagram of $\mathbb{Z}^2 \backslash S = \text{Vor}^{\sqcap}(S)$



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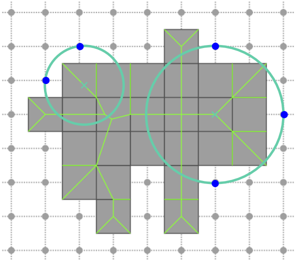

Set of open balls $\mathscr{B}$

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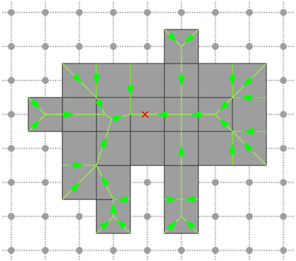

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If S has no hole, then $\mathrm{Vor}^{\sqcap}(S)$ is a tree $\mathcal{T}$.
▶ partial order $\leq_{\mathcal{T}}$ on $\mathscr{B}$ by picking a sink

Is $\mathscr{B}$ a good set of ranges ?

Sufficient conditions

If:

1. there exists a partial order $\leq_{\mathcal{T}}$ such that $(\mathscr{B}, \leq_{\mathcal{T}})$ is **anti-arborescent** ✓
2. $\forall x \in S$, the set $\{b \in \mathscr{B}, x \in b\}$ admits a **maximum** according to $\leq_{\mathcal{T}}$
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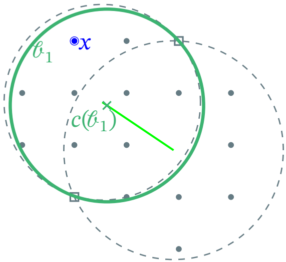

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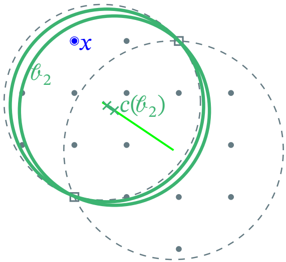

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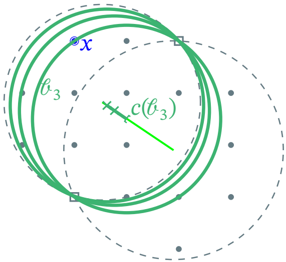

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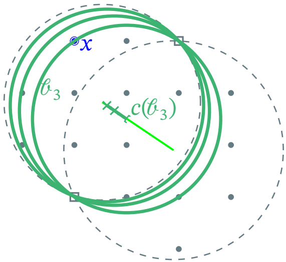

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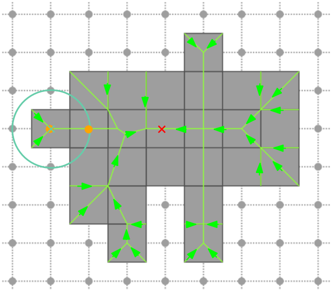

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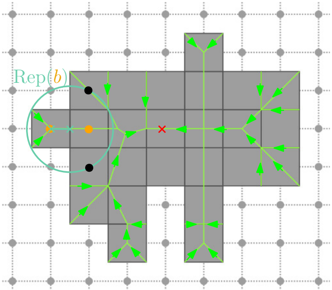

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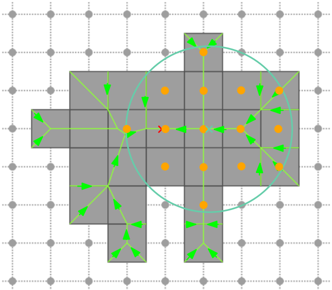

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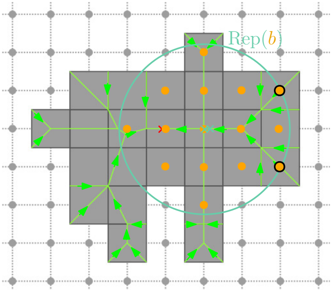

Property

$(\mathcal{B}, \leq_T)$ is a poset and is anti-arborescent

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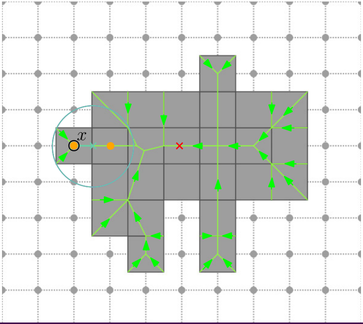

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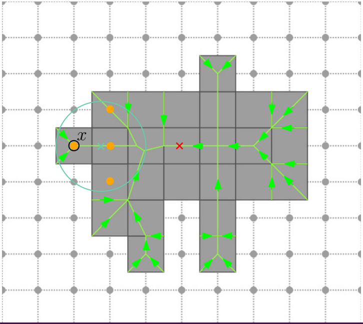

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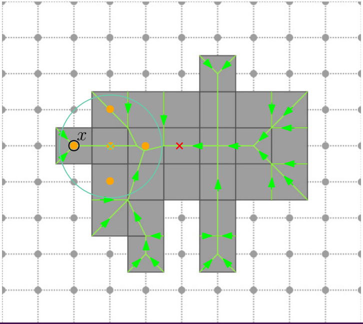

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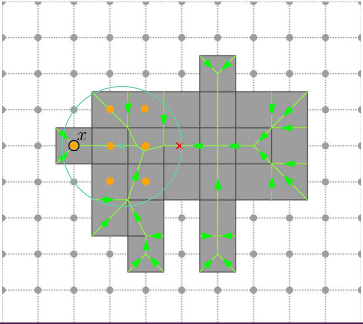

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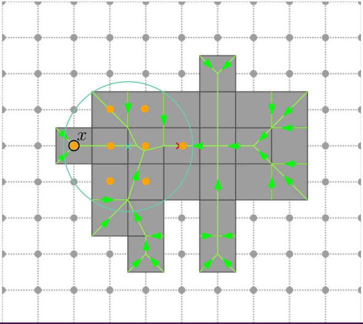

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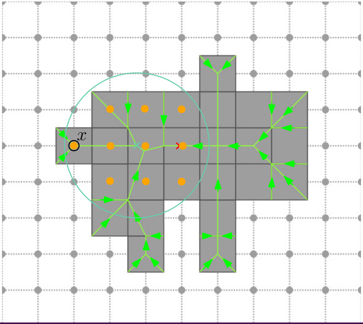

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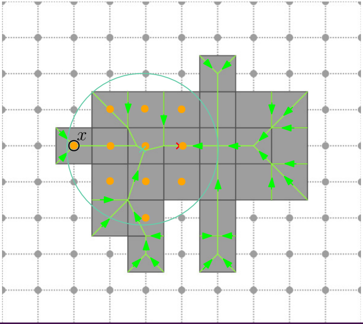

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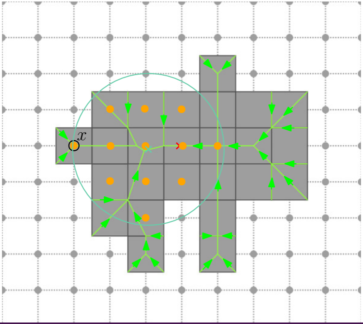

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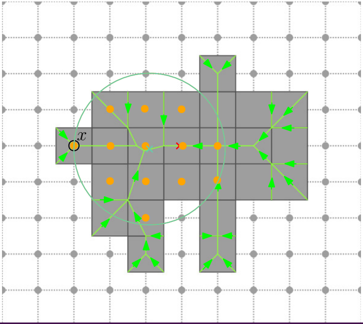

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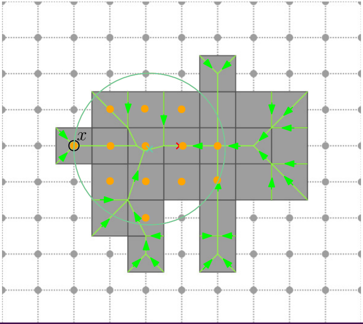

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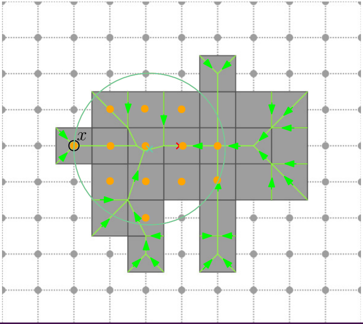

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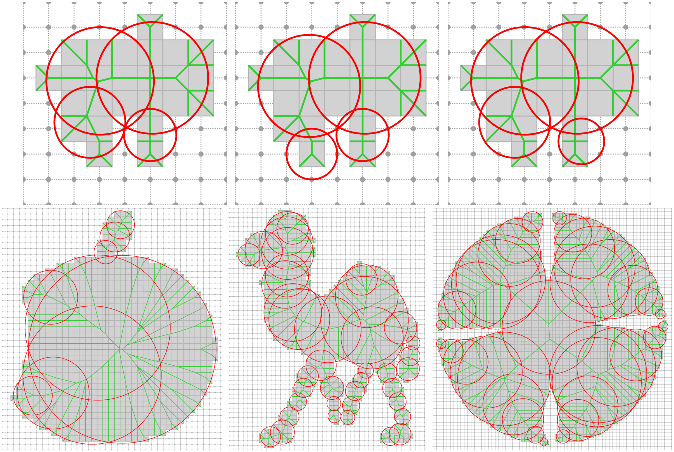
Some results

Implementation using DGtal (digital sets), CGAL (Voronoi diagram, disks) and Boost Graphs (topological order).



Thank you !